ACTIVE APPLICATION ORIENTED LEARNING OF COMPLEX DYNAMICAL SYSTEMS WITH APPLICATION TO MPC

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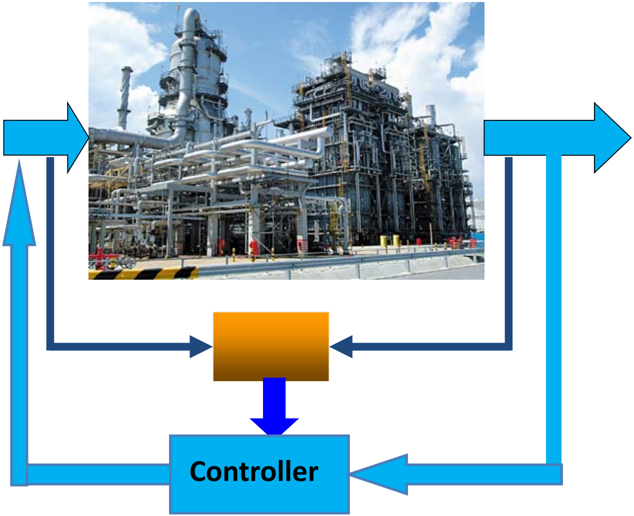
The problem



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I'm afraid this still describes state-of-the art....

Outline

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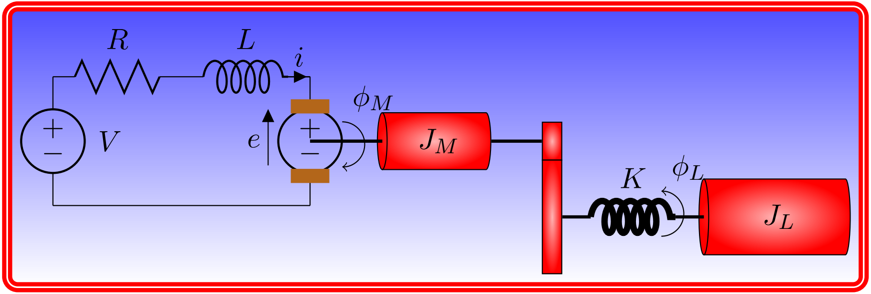
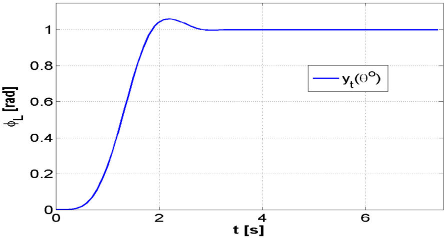
An application example: MPC of a DC-motor



- Input: Voltage V
- Output: Angle ϕ_L
- Model parameters θ: Resistance R, Moment of inertia J_L, Elasticity K, ...
- True parameters: θ_o

An application example: MPC of a DC-motor

- Ideal response: $y_t(\theta_o)$ - true parameters used in MPC



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- Ideal response: $y_t(\theta_o)$ - true parameters used in MPC
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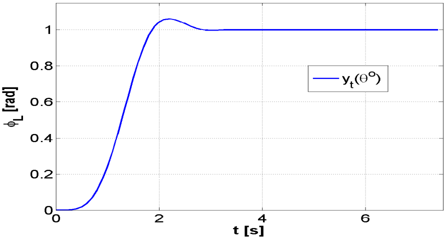

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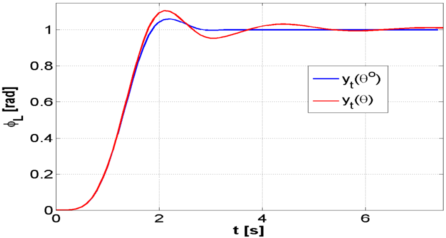

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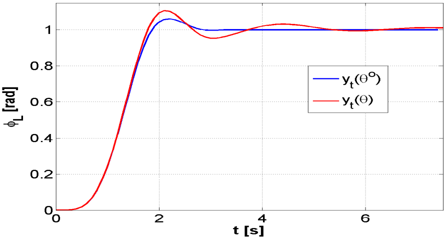

Performance degradation /Set of acceptable models

$$V_{\text{app}}(\theta) = \frac{1}{N}\sum_{t=1}^{N}(y_t(\theta_o) - y_t(\theta))^2$$

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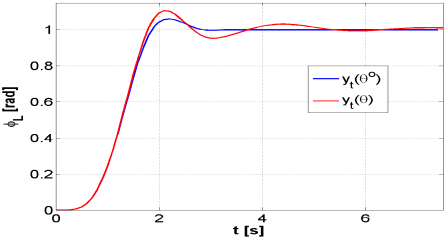

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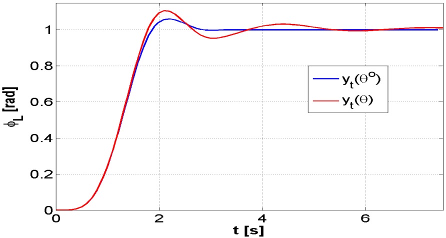

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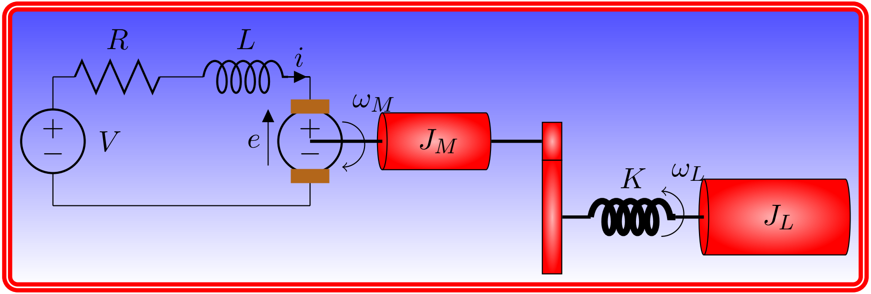

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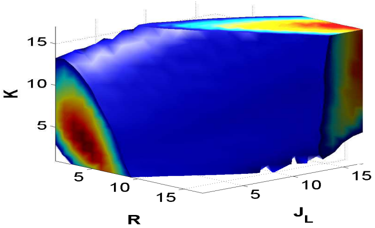

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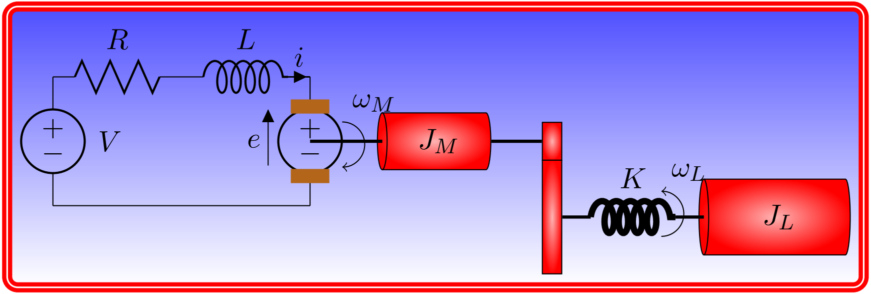

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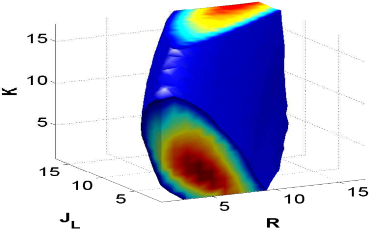

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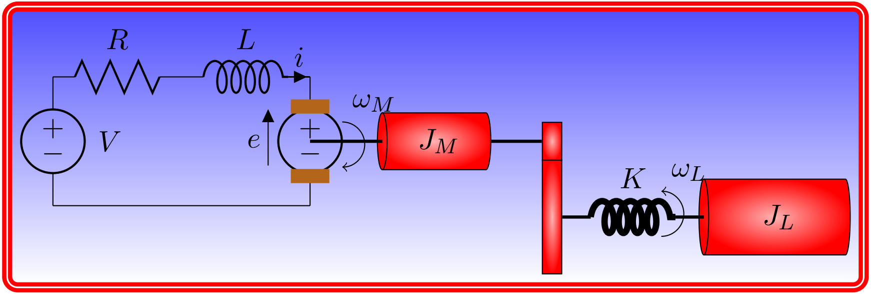

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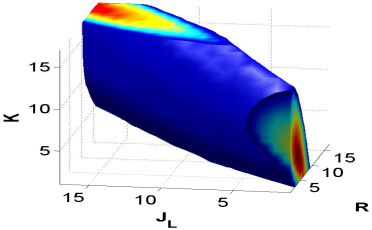

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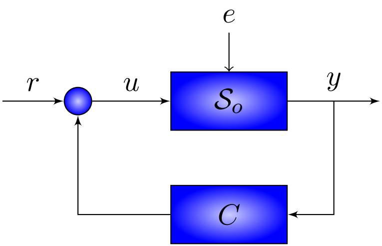

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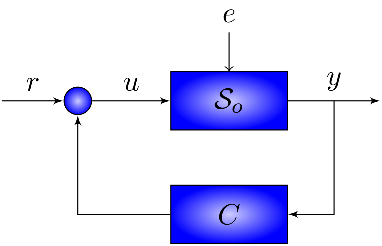

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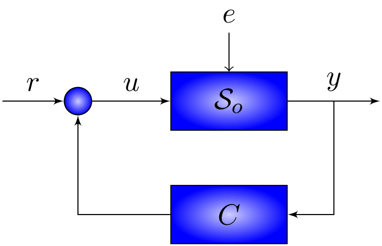

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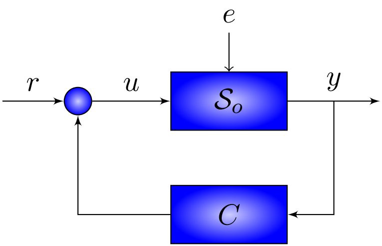

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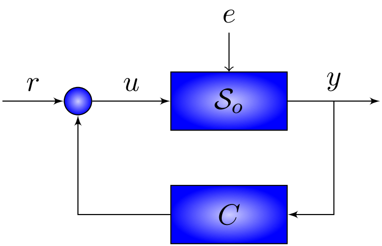

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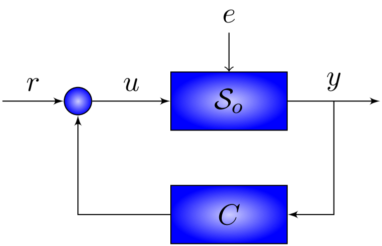

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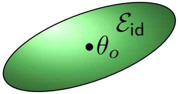

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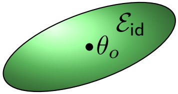

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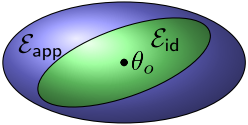

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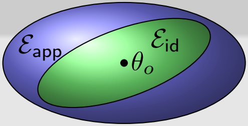

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can approximately be formulated as

Application oriented experiment design

$$\min N\mathrm{E}[u_t^2]$$

$$\text{s.t. } NV_{\mathsf{id}}(\theta) \geq \lambda_e \gamma n V_{\mathsf{app}}(\theta), \ \forall \theta \in \mathcal{E}_{\mathsf{app}}$$

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&= \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_u^{\mathsf{id}}(e^{j\omega}) \left| G_o(e^{j\omega}) - G(e^{j\omega},\theta) \right|^2 \mathrm{d}\omega
\end{aligned}
$$

Output error models

Application oriented experiment design

$$\min N\mathrm{E}[u_t^2]$$

s.t. $$\underbrace{NV_{\mathsf{id}}(\theta)}_{\frac{1}{2\pi}\int_{-\pi}^{\pi} N\Phi_u^{\mathsf{id}}(e^{j\omega})\,|G_o(e^{j\omega})-G(e^{j\omega},\theta)|^2\,\mathrm{d}\omega} \geq \lambda_e \gamma n V_{\mathsf{app}}(\theta), \ \forall\theta\in\mathcal{E}_{\mathsf{app}}$$

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- Minimization with respect to energy density spectrum $N\Phi_u^{\mathsf{id}}$
- Optimization tries to achieve

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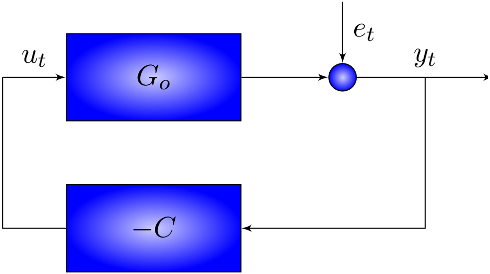
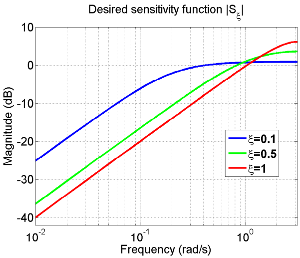
- Minimization with respect to energy density spectrum $N\Phi_u^{\mathsf{id}}$
- Optimization tries to achieve

$$NV_{\mathsf{id}}(\theta) = \lambda_e\,\gamma\,nV_{\mathsf{app}}(\theta),\ \forall\theta\in\mathcal{E}_{\mathsf{app}}$$

Identification cost matched to performance degradation

Model Reference Control





- Controller $C = C(G)$, G output error model

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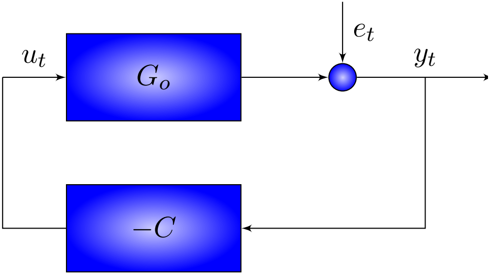

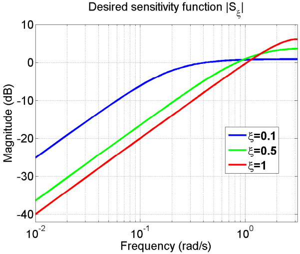

- Controller $C = C(G)$, G output error model
- Desired sensitivity function: S_ξ

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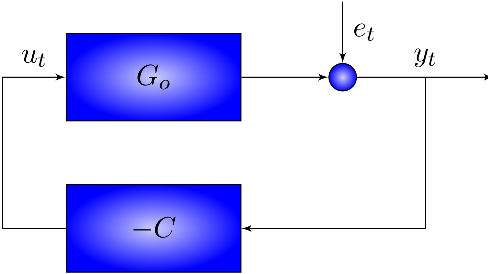

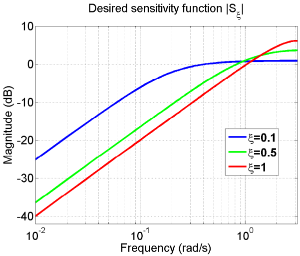

- Controller $C = C(G)$, G output error model
- Desired sensitivity function: S_ξ
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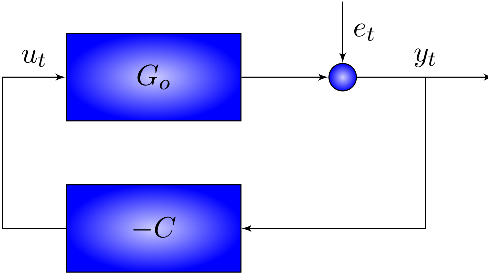
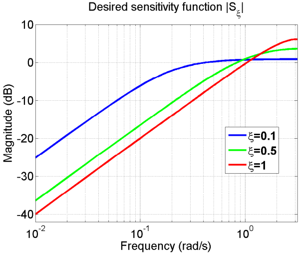
Model Reference Control





- Controller $C = C(G)$, G output error model
- Desired sensitivity function: S_ξ
- Achieved sensitivity function: $S(G) = \frac{1}{1+C(G)G_o}$
- Performance degradation: $V_{\text{app}}(G) := \left\| \frac{S(G)-S_\xi}{S_\xi} \right\|_2^2$

Model Reference Control

$$\min N\mathrm{E}[u^2(t)]$$
$$\text{s.t. } NV_{\mathsf{id}}(\theta) \geq \gamma\lambda_e n\, V_{\mathsf{app}}(\theta)$$

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- *Experimental conditions during identification should be a scaled version of the desired operating conditions!*

Outline

Static gain estimation

$$y_t = \sum_{t=1}^{n} \theta_k u_{t-k} + e_t$$

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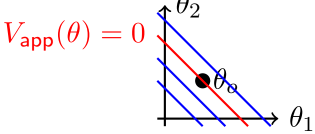

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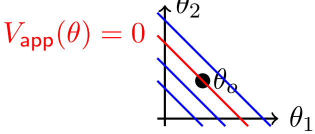

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Static gain estimation

Model order:	low	true	high
Accuracy:		*good*	

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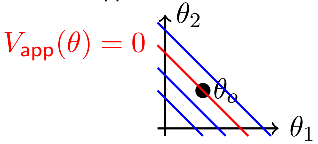

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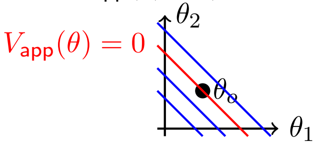

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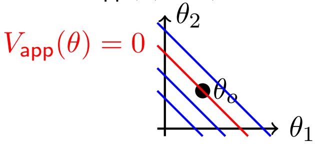

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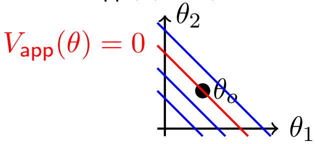

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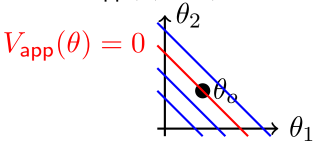

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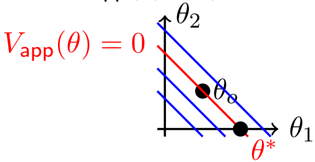

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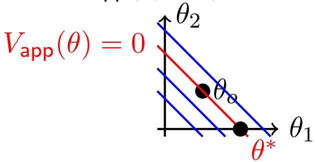

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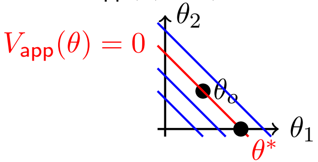

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Application oriented experiment design: Summary

Application oriented experiment design

Aims at achieving

$$N\,V_{\mathsf{id}}(\theta) = \lambda_e\,\gamma\,n\,V_{\mathsf{app}}(\theta)$$

using minimum energy

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(The let sleeping dogs lie paradigm)

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- As a result, the entire system may not have to be identified!
  - Choice of model structure less critical

Outline

Computations: The Information Application Inequality

The design constraint can be written as

$$\mathcal{E}_{\mathsf{id}} \quad \subseteq \quad \mathcal{E}_{\mathsf{app}},$$

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The Information Application Inequality

Recall: V_{id} linear in the input spectrum

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The Information Application Inequality

Recall: V_{id} linear in the input spectrum

Information Application Inequality is an LMI in the input spectrum

Computations

$$
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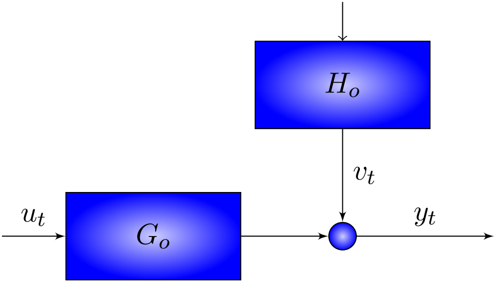

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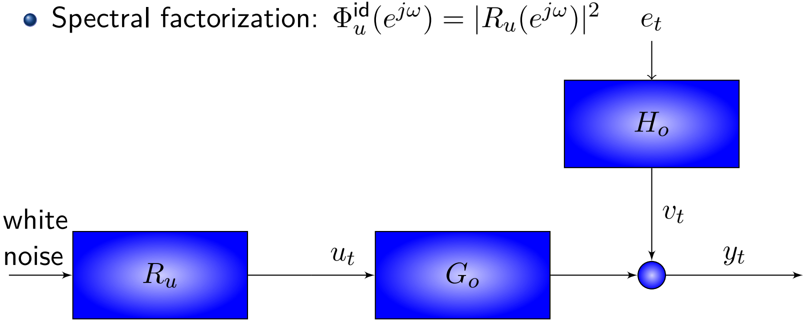
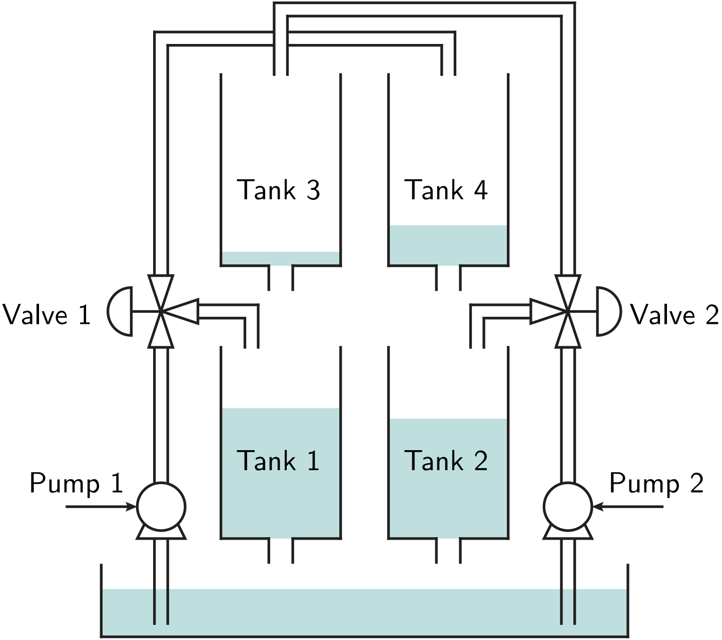


Outline

Experimental results: Water tank process



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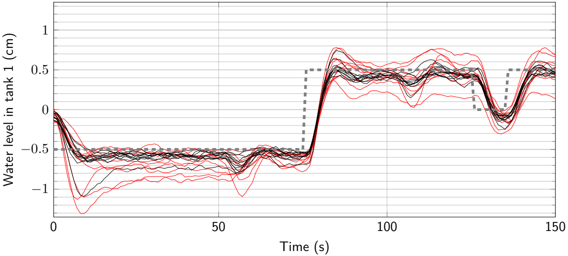
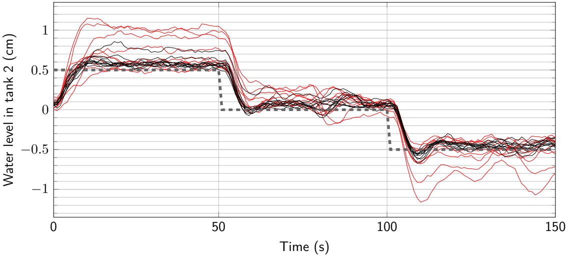


(a)



(b)

MPC: Black: based on AOID-model. Red: based on white noise excitation

Computations

Application oriented experiment design

$$\min N \mathrm{E}[u^2(t)]$$

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- Optimization problem depends on the unknown system!
- Solutions:
  - Robust experiment design
    (e.g. Rojas, Welsh, Goodwin, Feuer 2007)

Computations

Application oriented experiment design

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$$\text{s.t. } \mathcal{I}_1^N(\theta_o) \succeq \frac{\gamma n}{2} V''_{\mathsf{app}}(\theta_o)$$

- Optimization problem depends on the unknown system!
- Solutions:
  - Robust experiment design
    (e.g. Rojas, Welsh, Goodwin, Feuer 2007)
  - Adaptive (or sequential) experiment design

Computations

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Outline

Application oriented experiment design

Output error models

The impact of optimal experiments on the identification problem

Computing the optimal input

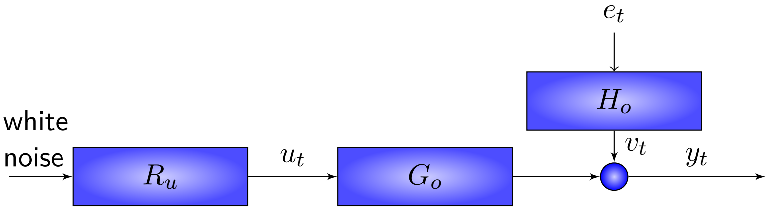
Experimental results

Active application oriented learning

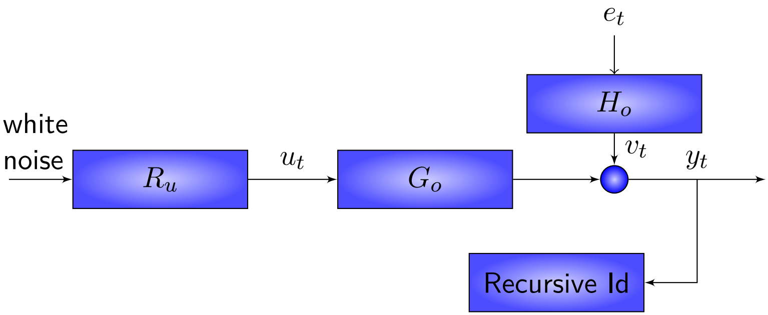
Application oriented dual control

Summary

Active application oriented learning



Active application oriented learning



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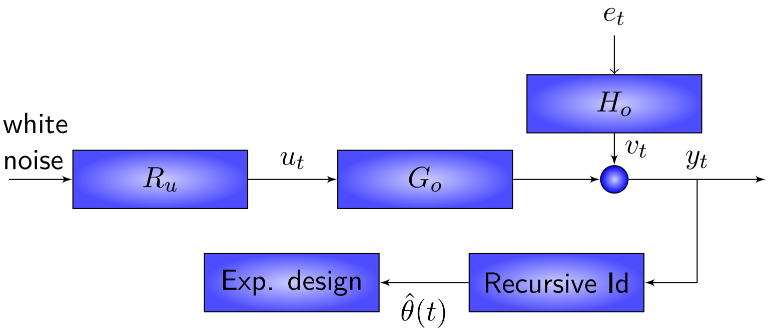

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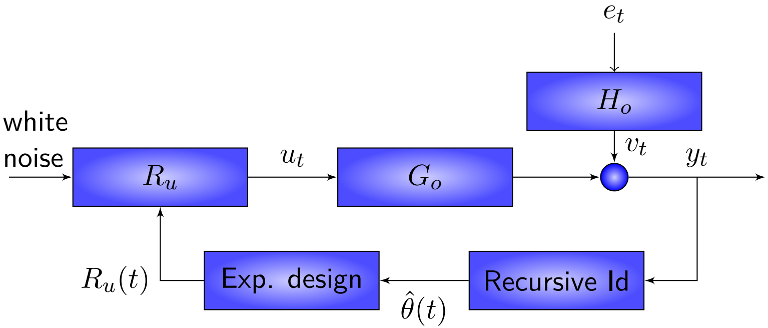

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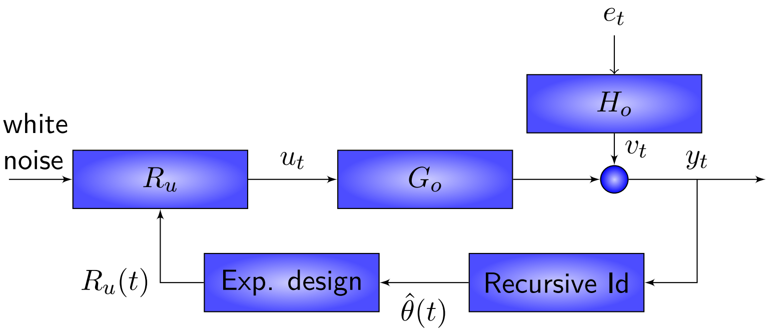
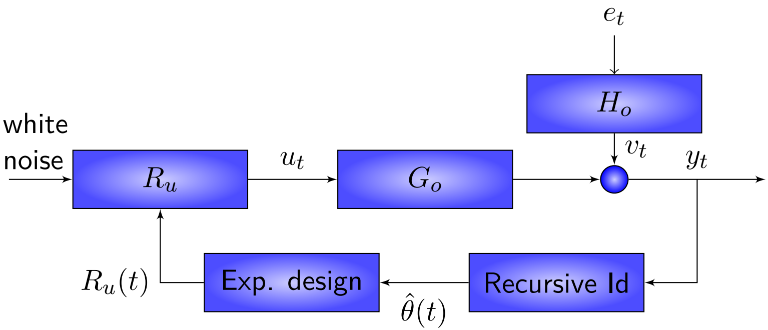


- An adaptive feedback system

Active application oriented learning



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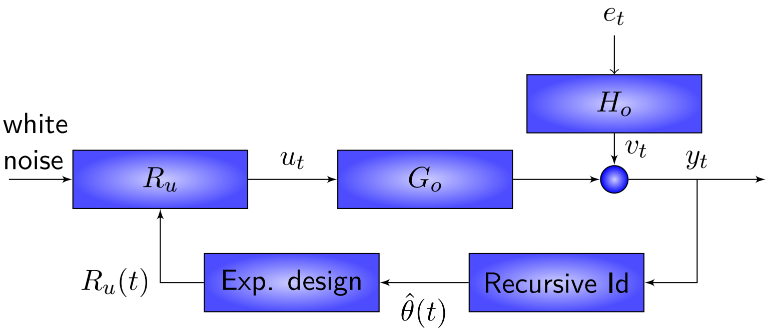

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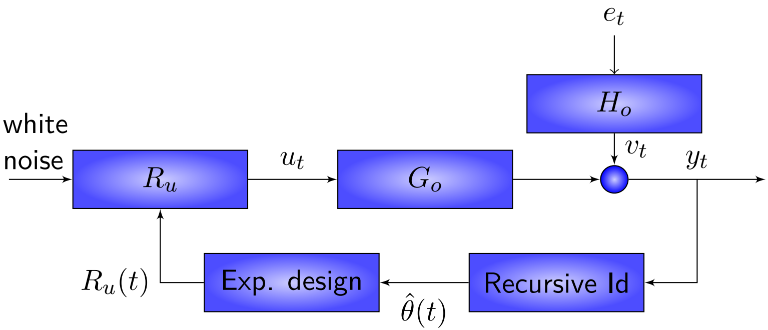

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Key questions:

- Convergence?
- Accuracy?

Active application oriented learning

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Theorem

- *True linear time-invariant system in the model set*
- *System stable*

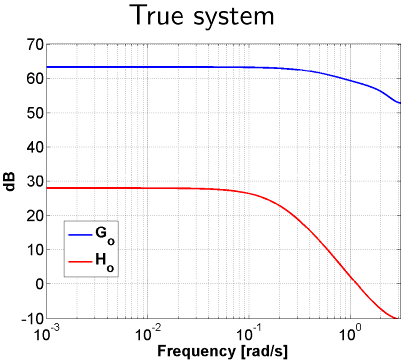
$\Rightarrow$ $\hat{\theta}(t)$ *has the same asymptotic accuracy as the off-line estimate that uses data collected under the optimal experimental conditions (using knowledge of* θ_o*)*

Active application oriented learning

What happens when true system is not in the model set?

Example: Non-minimum phase zero estimation

True system: $y_t = \dfrac{(q-3)(q-0.1)(q-0.2)(q+0.3)}{q^4(q-0.5)} u_t + \dfrac{q}{q-0.8} e_t^o$



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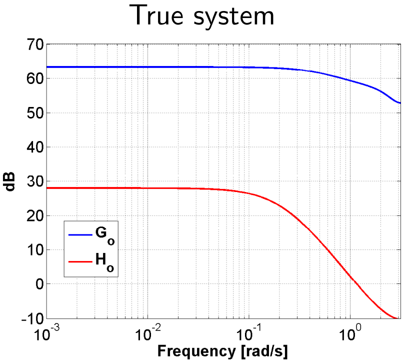

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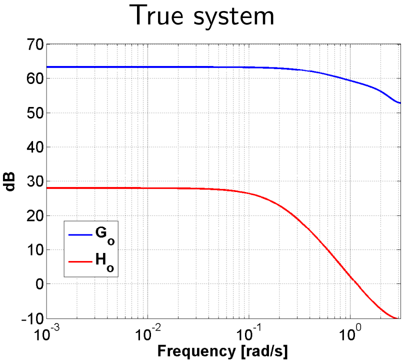

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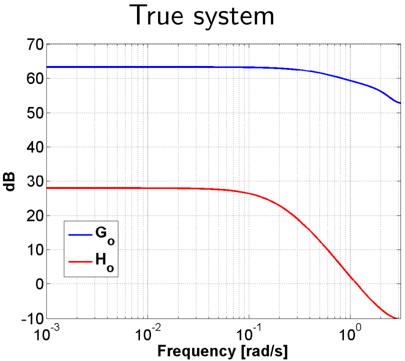

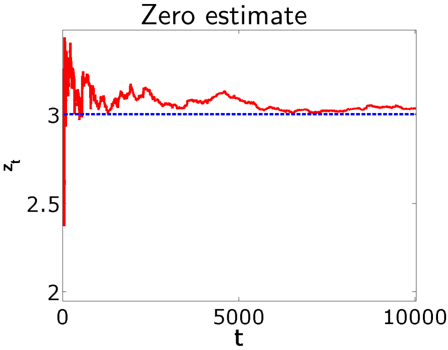

Outline

Application oriented dual control

$$\min_{\text{Input spectrum}} \quad N\mathrm{E}[u_t^2], \quad \text{s.t. } \mathcal{I}_1^N(\theta_o) \succeq \frac{\gamma n}{2} V''_{\mathsf{app}}(\theta_o)$$

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- Receeding horizon formulation: MPC-X

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Policy: $\pi_t(x, u) = \mathbb{P}\{u_t = u \,|x_t = x\}$

Markov Decision Process formulation: Implementation

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Define z_{xu} as the probability of being in state x and taking action u

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Occupancy measure

MDP problem is a semi-definite program in $\{z_{xu}\}$.

Markov Decision Process formulation: Simulation study

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x split in 51 regions.

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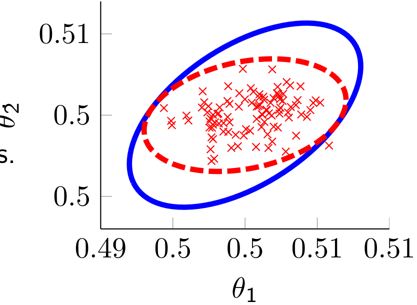

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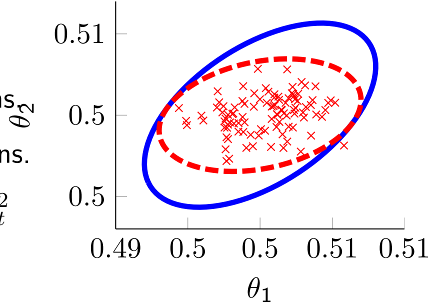

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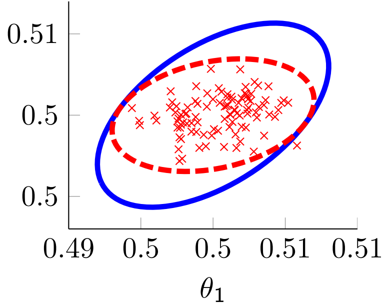

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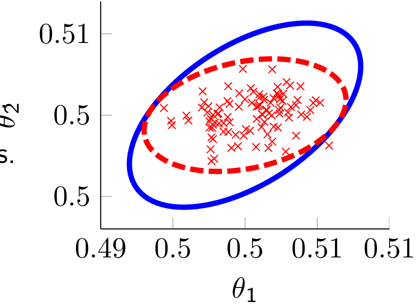

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Crosses: 100 Monte Carlo simulations using the MDP controller

Markov Decision Process formulation: Summary

- Elegant and powerful formulation

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- Leads to a semi-definite program

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- Elegant and powerful formulation
- Leads to a semi-definite program
- but suffers from the curse of dimensionality due to discretization of state-space

Receeding horizon formulation

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Cost at time t:

$$C_t = \sum_{k=1}^{F} c_t(x_k, u_k) = \sum_{k=1}^{F} \|y_{k+1} - r_{t+k+1}\|_Q^2 + \sum_{k=1}^{F} \|u_k\|_S^2$$

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Major problems:

- θ_o unknown
- *Data not stationary & feedback*

  $\Rightarrow$ *Cannot use spectrum as design variable*

Receeding horizon formulation: Implementation

Approximations:

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- Initial estimate $\hat{\theta}$ replaces θ_o

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MPC-X: Model Predictive Control with eXperimental constraints

Receeding horizon formulation: Alternative approaches

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- Multiobjective MPC with identification (Aggelogiannaki and Sarimveis (2006)): $P = 0$
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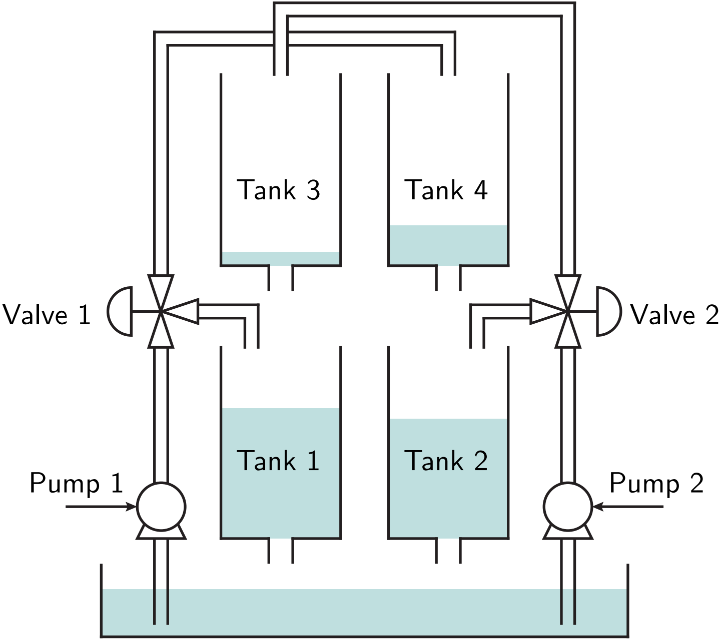
Do not take application into account explicitly

Receeding horizon formulation: Simulation study



Receeding horizon formulation: Simulation study

$$\begin{cases} x_{t+1} = \begin{bmatrix} \theta_3 & \theta_4 \\ 1 & 0 \end{bmatrix} x_t + \begin{bmatrix} 4.5 \\ 0 \end{bmatrix} u_t, \\ \end{cases}$$

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$\theta_o = \begin{bmatrix} 0.12 & 0.059 & 0.74 & -0.14 \end{bmatrix}^T$ Noise var.: 0.01.
$N = 200,\ F = 5$

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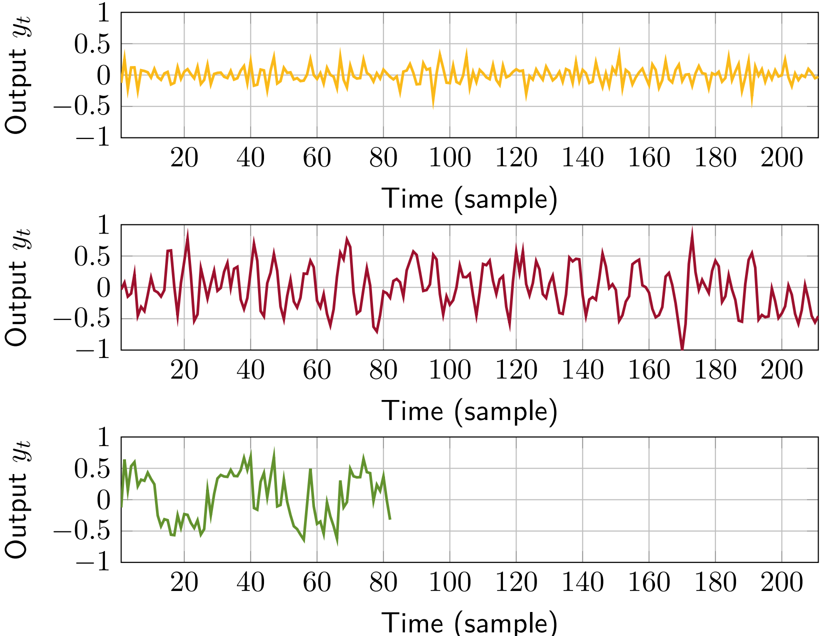
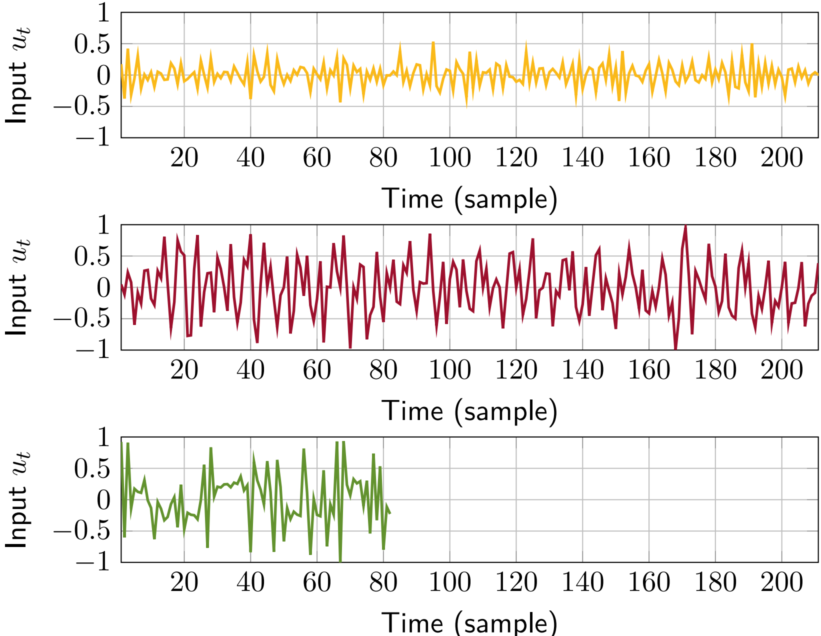
MPC-X: Minimum time formulation

Receeding horizon formulation: Simulation study



Receeding horizon formulation: Simulation study



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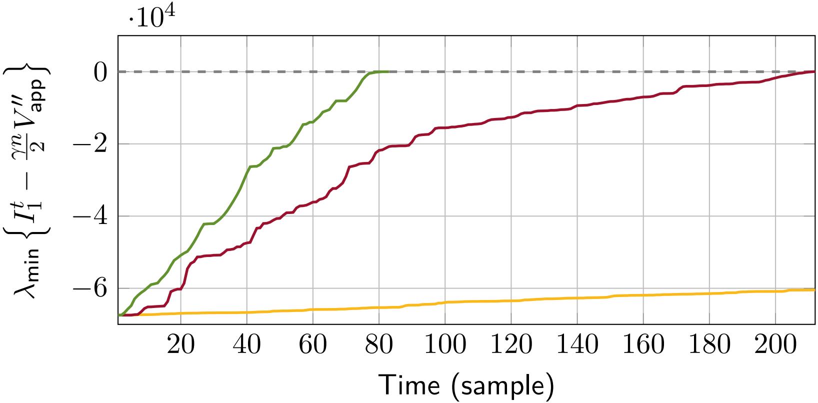
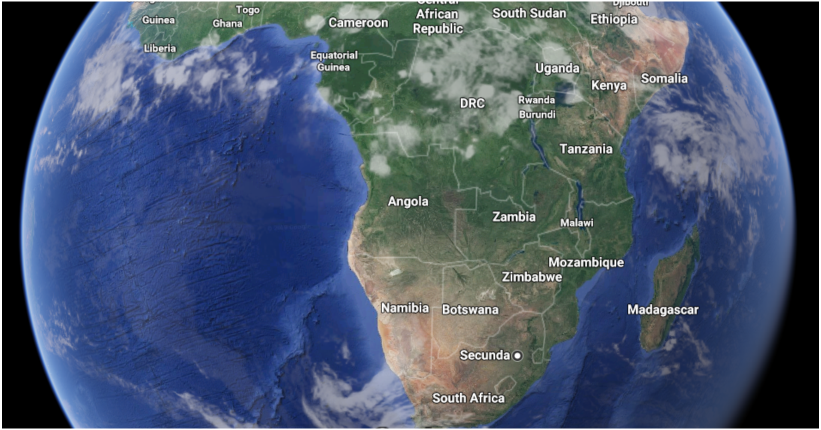


- Regular MPC (—),
- PE-MPC with $\rho = 0.5$ (—)
- Minimum time MPC-X (—)

Receeding horizon formulation: Simulation study

Algorithm	$\mathrm{Var}\, u$	$\mathrm{Var}\, y$	N
MPC-X, minimum time	0.203	0.146	82
PE-MPC, $\rho = 0.5$	0.175	0.120	211

MPC-X experimental study: Let's travel

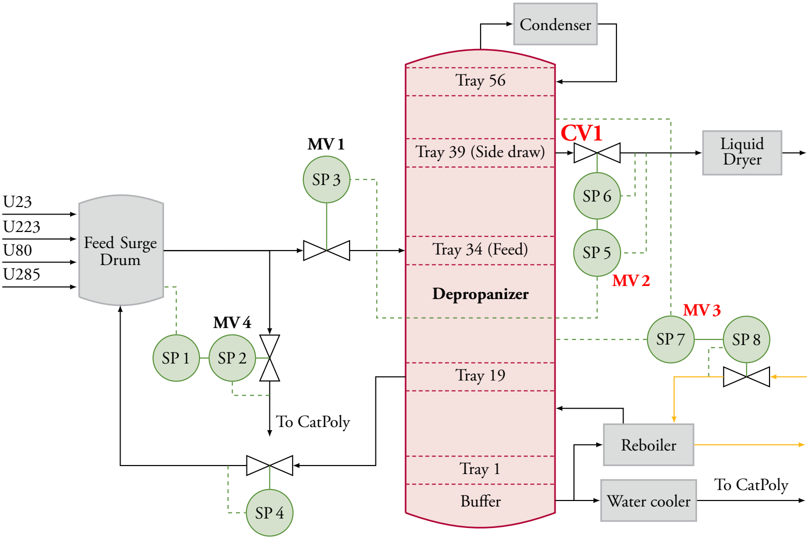


Secunda, South Africa

SASOL Synthetic Fuels Refinery

Synfuels Catalytic Cracker (SCC)

Depropanizer



Depropanizer

Separates three-carbon hydrocarbons (C_3) from four carbon hydrocarbons (C_4)

Objective: Set point for CV1=C_4 concentration in side draw

MV2: Side draw to feed ratio

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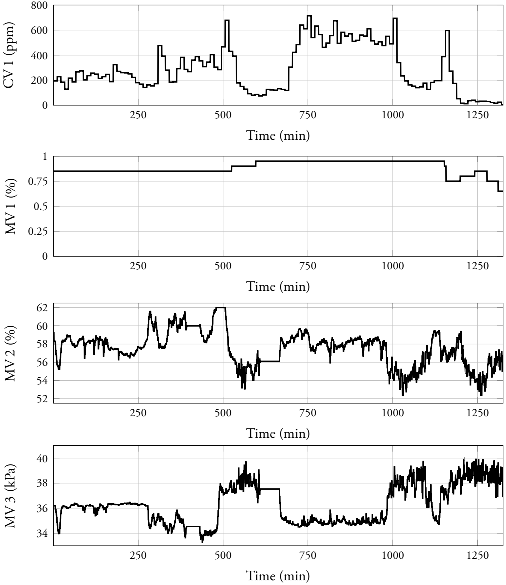
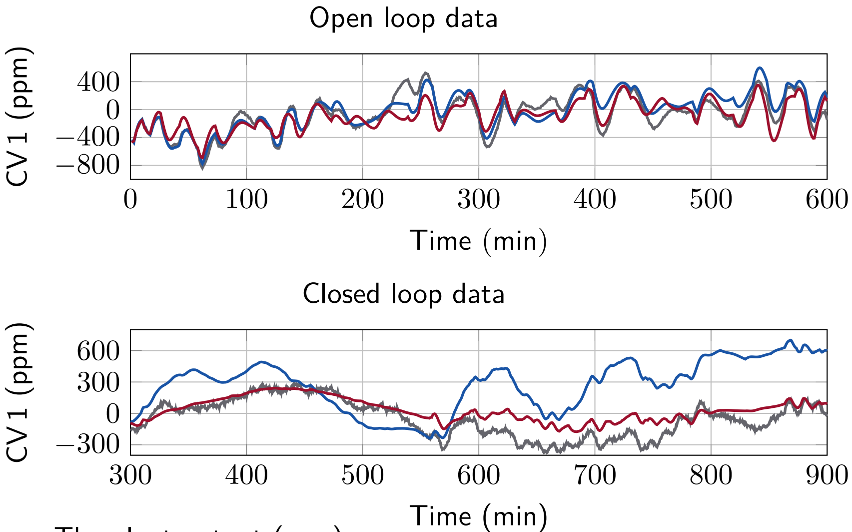
Performance drop obtained by changing poles of model

Excitation level manually controlled

Depropanizer: MPC-X experiment



Depropanizer: Model fit



- The plant output (——)
- Model identified in open-loop (——)
- Model identified in closed-loop MPC-X experiment (——)

Depropanizer: Closed loop performance

	Variance	
Model	CV 1	MV 5
Before MPC-X	95×10^3	34×10^7
After MPC-X model update	36×10^3	37×10^7

MV 5 $= C_4$ content in the feed

MPC-X: Summary

- Sample version of Information Application Inequality added as a matrix inequality constraint in MPC

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- Convex relaxation

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- Current limitation: Output error models (disturbances not modeled)

Outline

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What have we learnt?

- A framework for experiment design where the application is taken into account
- The optimal experiment matches the identification criterion to the performance degradation using parsimonious excitation
  (The let sleeping dogs lie paradigm)
- Simplifies the identification problem
- Active application oriented learning practical implementation
- Adding the Information Application Inequality to an optimal control problem leads to dual control

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- Former PhD-students: Kristian Lindqvist, Henrik Jansson, Jonas Mårtensson, Märta Barenthin, Christian Larsson, Afrooz Ebadat, Mariette Annergren

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THANK YOU!!!